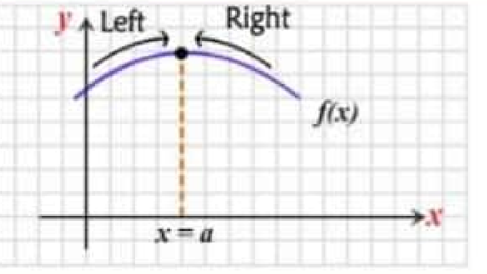
CONTINUITY

Part I

Mathematical Definition of Point Continuity

A function f(x) is said to be continuous at x = a iff,

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a) = \text{finite quantity}$$



One sided Continuity

- continuous at x=a from left if $\lim_{x\to a^-} f(x) = f(a)$
- continuous at x=a from right if $\lim_{x\to a^+} f(x) = f(a)$

Continuity In An Interval

In an Open Interval (a,b) if

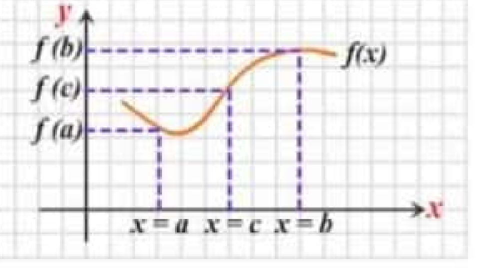
If it is continuous at each and every point $c \in (a,b)$.

In a Closed Interval [a,b] if

(a) f(x) is continuous in (a,b).

(b) f(x) is right continuous at x=a. i.e. $\lim_{x\to a^+} f(x) = f(a)$ = a finite quantity

(c) f(x) is left continuous at x=b. i.e. $\lim_{x\to b^-} f(x) = f(b)$ = a finite quantity



Theorems of Continuity

Theorem - 1 If 'f' & 'g' are continuous at x = a, then $f \pm g$, f.g will also be continuous at x = a. And $\frac{f}{g}$ will also be continuous provided $g(a) \neq 0$

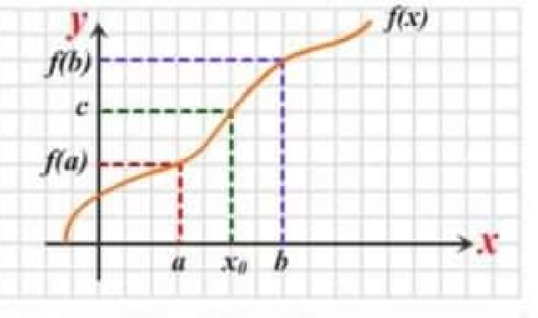
Theorem - 2 If 'f' is continuous at x = a & 'g' is discontinuous at x = a then $f \pm g$, must be discontinuous at x = a. However nothing definite can be said about f.g or f/g.

Theorem - 3 If f(x) & g(x) are discontinuous at x = a then nothing definite can be said about $f \pm g$, f.g or f/g

Theorem - 4

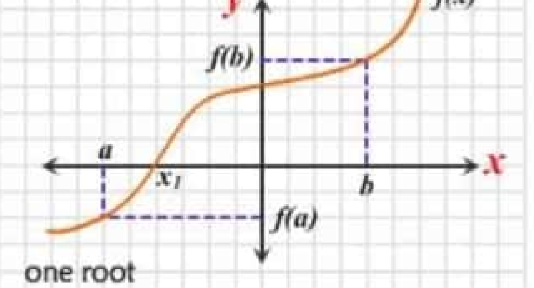
Intermediate value theorem

If 'f' is continuous on [a, b] & $f(a) \neq f(b)$ then for any value $c \in (f(a), f(b))$ there exists at least one number $x_0 \in (a, b)$ such that $f(x_0) = c$

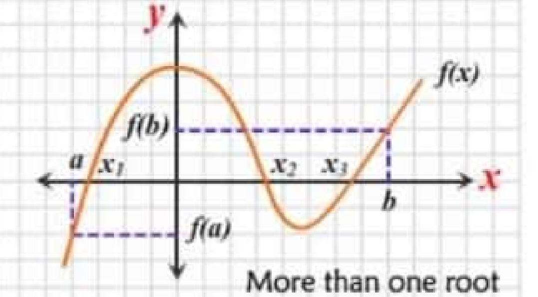


Alternatively

f(x) is continuous in [a,b] and f(a) & (b) have opposite signs then the equation f(x) = 0 has at least one root in (a,b).



one root



More than one root